ME-221
PROBLEM SET 6

Problem 1

Determine the Laplace transform of the following signals:

$$f_1(t) = \begin{cases} 0 & t < 0 \\ Asin(wt) & 0 \leq t < \pi/w \\ 0 & t \geq \pi/w \end{cases}$$

$$f_2(t) = \begin{cases} 0 & t < 0 \\ 1 + e^{\alpha t} & 0 \leq t < T \\ -e^{\alpha t} & t \geq T \end{cases}$$

$$f_3(t) = tsin(wt) \qquad t \geq 0$$

Problem 2

Consider the mechanical system shown in Figure 1. The system is initially at rest. The displacements $x_1(t)$ and $x_2(t)$ are measured with respect to their equilibrium positions before the application of an external force $F(t)$. The spring and the viscous damping coefficients are given by k and f, respectively.

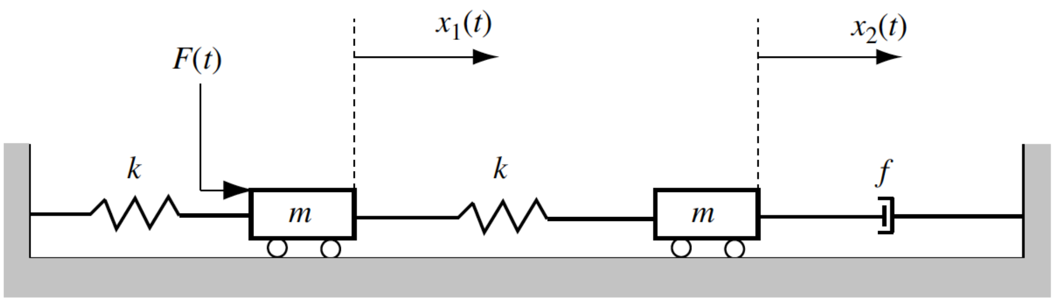

Figure 1: Mechanical System

(a) Calculate the transfer function G(s) by taking $F(t)$ as the input and $x_2(t)$ as the output of the system.

(b) Propose an analogous electrical circuit.

(c) What is the order of the system? How many state variables would you define for deriving a state-space representation of the system?

Problem 3

(a) Derive the Laplace transform of the function $f(t) = Asin(\omega t + \phi)$ with $t >= 0$. Hint: Use a trigonometric identity to find an expression that can be easily transformed.

(b) Given that $F(s) = \frac{s+6}{s^2+12}$, obtain the values of A, ω, and ϕ where $A > 0$.

(c) Generalize the Laplace transform for functions in the form of $Ae^{-at}sin(\omega t + \phi)$ with $t >= 0$.

Problem 4

Consider a system described by the following differential equation

$$\dddot{y}(t) + 8\ddot{y}(t) + 17\dot{y}(t) + 10y(t) = 0$$

where the initial conditions are given as $y(0) = 2, \dot{y}(0) = 1, \ddot{y}(0) = 0.5$.

(a) Calculate $Y(s)$, the Laplace transform of $y(t)$.

(b) Validate the values of $y(0)$ and $\dot{y}(0)$ using the initial value theorem.

Problem 5

Find the equivalent state-space representation of the system described by the following transfer function. The system is initially at rest.

$$G(s) = \frac{s+1}{s^2+s+2}$$